Kobayashi, Shin-ichi

Iwasawa theory for elliptic curves at supersingular primes. (English) Zbl 1047.11105

Invent. Math. 152, No. 1, 1-36 (2003).

Let p be an odd prime, $\mathbb{Q}_\infty = \bigcup_n F_n$ the cyclotomic $\mathbb{Z}_p$-extension of $\mathbb{Q}$, Λ the usual Iwasawa algebra. In the Iwasawa theory of elliptic curves at good ordinary primes, the Main Conjecture states that the Selmer group over $\mathbb{Q}_\infty$ is Λ-cotorsion and the characteristic ideal is generated by the Mazur and Swinnerton-Dyer p-adic L-function. But at supersingular primes, the Selmer group is no longer cotorsion and the p-adic L-function does not live in $\Lambda \otimes \overline{\mathbb{Q}}_p$.

In this paper, the author proposes a new formulation at good supersingular primes by modifying both the Selmer group and the p-adic L-function. More precisely, let E be an elliptic curve over $\mathbb{Q}$ with good supersingular reduction at p, and such that $a_p = 0$ (this is automatically satisfied for a supersingular $p > 3$). The modified Selmer groups $\mathrm{Sel}^{\pm}(E/F_n)$ are defined by replacing the groups of local points by the modified groups $E^{\pm}(F_{n,p})$ introduced by *B. Perrin-Riou* [Invent. Math. 99, No. 2, 247–292 (1990; Zbl 0715.11030)] and *H. Knospe* [Manuscr. Math. 87, 225–258 (1995; Zbl 0847.14026)]. The relevant analytical objects are Pollack's p-adic L-functions $\mathcal{L}_p^{\pm}(E, X)$, which interpolate special values of the Hasse-Weil L-function and live in Λ. The author shows that the modified Selmer groups $\mathrm{Sel}^{\pm}(E/\mathbb{Q}_\infty)$ are Λ-cotorsion, and he conjectures that $\mathrm{char}\,(\mathrm{Sel}^{\pm}(E/\mathbb{Q}_\infty)^\vee) = (\mathcal{L}_p^{\pm}(E, X))$. He shows that this supersingular Main Conjecture (say SSMC) is equivalent to the conjecture formulated in cohomological terms (for general motives) by *K. Kato* [Arithmetic algebraic geometry. Lect. Notes Math. 1553, 50–163 (1993; Zbl 0815.11051)] and Perrin-Riou (loc. cit.)].

The SSMC was recently proved in the CM case by *R. Pollack* and *K. Rubin* [Ann. Math. (2) 159, No. 1, 447–464 (2004; Zbl 1082.11035)]. Here, in the non-CM case, the author shows half of the SSMC, namely that $\mathcal{L}_p^{\pm}(E, X)$ lives in the corresponding characteristic ideal. He also derives an asymptotic formula for the p-adic order of the Tate-Shafarevich groups $Ш(E/F_n)$ (assuming their finiteness) involving the λ-and μ-invariants of $\mathrm{Sel}^{\pm}(E/F_\infty)^\vee$. This improves upon similar formulas obtained previously by Kurihara and Perrin-Riou with unspecified rational numbers λ and μ; here, the invariants are specified as $\lambda_\pm$ and $\mu_\pm$ as in Pollack's analytic counterpart.

The key point is the construction of Λ-valued $\pm$ Coleman maps which send Kato's zeta element to Pollack's p-adic L-function. Once these maps are constructed, the proofs proceed in the same way as in the good ordinary case.

Reviewer: Thong Nguyen Quang Do (Besançon)

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