Chen, Zongxuan

On the hyperorder of solutions of higher order differential equations. (English)

Zbl 1047.30019
Chin. Ann. Math., Ser. B 24, No. 4, 501-508 (2003).

This paper is devoted to proving that under certain very special cases, all transcendental solutions f of the differential equation

$$f^{(k)} + H_{k-1}(z)f^{(k-1)} + \cdots + H_1(z)f' + H_0(z)f = 0$$

are of infinite order of growth. In the main result, the assumptions are as follows: Each $H_j(z) = h_j(z)e^{\alpha_j z}$, where h_j is a polynomial and $\alpha_j \in \mathbb{C}$. Moreover, at least for some $s < l$, h_s and h_l are nonvanishing, and $\alpha_s = d_s e^{i\varphi}$, $\alpha_l = -d_l e^{i\varphi}$, where $d_s > 0$, $d_l > 0$. In addition, for $j \neq s$, either $\alpha_j = d_j e^{i\varphi}$, or $\alpha_j = -d_j e^{i\varphi}$ for $d_j \geq 0$, and $\max\{\, d_j \mid j \neq s, l \,\} =: d < \min\{d_s, d_l\}$. More precisely, it will be proved that under these conditions, the iterated order $\rho_2(f) := \limsup_{r\to\infty} \frac{\log\log T(r,f)}{\log r} = 1$. The proof makes use of basic results from the Wiman–Valiron theory as well as standard estimates for generalized logarithmic derivatives.

Reviewer: Ilpo Laine (Joensuu)

MSC:

30D35 Value distribution of meromorphic functions of one complex variable, Nevanlinna theory

Full Text: DOI

References:

[1] doi:10.1515/9783110863147· doi:10.1515/9783110863147