Duman, O.; Orhan, C.
Statistical approximation by positive linear operators. (English) Zbl 1049.41016
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The sequences of some classical approximation operators tend to converge to the values of the function they approximate, except perhaps at points of discontinuity, where in several cases such sequences do not converge to any value. Statistical convergence, which is a regular non-matrix summability method, has revealed effective to correct the lack of convergence. Using A-statistical convergence, the authors prove a Korovkin type approximation which concerns the problem of approximating a function f by means of a sequence (T_n) of positive linear operators acting from a weighted space C_{ρ_1} into a weighted space B_{ρ_2}. Let $A=(a_{jn})$ be an infinite summability matrix. For a given sequence $x=(x_n)$, the A-transform $Ax:=((Ax)_j)$ of x is given by $(Ax)_j=\sum_{n=1}^\infty a_{jn}x_n$, provided the series converges for each j. It is said that A is regular if $\lim_j (Ax)_j=L$ whenever $\lim_j x_j=L$. If A is a non-negative regular summabilitiy matrix and K is a subset of $\mathbf{N}:=\{1,2,\dots\}$, then the A-density of K is defined as $\delta_A(K)=\lim_j\sum_{n=1}^\infty a_{jn}\chi_K(n)$ provided the limit exists, where χ_K is the characteristic function of K. A sequence $x=(x_n)$ is said to be A-statistically convergent to a number L ($\text{st}_A\text{-}\lim x=L$) if, for every $\varepsilon>0$, $\delta_A(\{n\in\mathbf{N}\colon |x_n-L|\geq\varepsilon\})=0$. The concept of A-statistical convergence can be given for sequences (x_n) in a normed space. Let $\mathbf{R}$ denote the set of real numbers. A weight function is a continuous function $\rho:\mathbf{R}\to[1,+\infty)$ tending to ∞ as $|x|\to\infty$. In such a case, the weighted space B_ρ is the space of real-valued functions f defined on $\mathbf{R}$ satisfying $|f(x)|\leq M_f\rho(x)$ for all $x\in\mathbf{R}$, where M_f is a constant depending on f. The weighted subspace C_ρ is $C_\rho:=\{f\in B_\rho: f \text{ is continuous on } \mathbf{R}\}$. The spaces B_ρ and C_ρ are Banach spaces with the norm $\|f\|_\rho=\sup|f|/\rho$, where the supremum is taken over $\mathbf{R}$. A classical Korovkin type approximation theorem asserts the following: Assume that ρ_1, ρ_2 are weight functions satisfying $(*)$ $\lim_{|x|\to\infty}\rho_1(x)/\rho_2(x)=0$ and that $T_n:C_{\rho_1}\to B_{\rho_2}$ $(n\geq 1)$ is a sequence of positive linear operators. Then $\lim_n\|T_nf-f\|_{\rho_2}=0$ for all $f\in C_{\rho_1}$ if and only if $\lim_n\|T_nF_\nu-F_\nu\|_{\rho_1}=0$ for $\nu=0,1,2$, where $F_\nu(x):=x^\nu\rho_1(x)/(1+x^2)$. The authors prove an extension of the last theorem with the ordinary limit operator replaced by an A-statistical limit operator. Specifically, they demonstrate the following. Let $A=(a_{jn})$ be a non-negative regular summability matrix and let (T_n) as before, where the weight functions ρ_1, ρ_2 satisfy $(*)$. Then $\text{st}_A\text{-}\lim_n\|T_nf-f\|_{\rho_2}=0$ for all $f\in C_{\rho_1}$ if and only if $\text{st}_A\text{-}\lim_n\|T_nF_\nu-F_\nu\|_{\rho_1}=0$ for $\nu=0,1,2$. If, in addition, φ is a continuous increasing function on $\mathbf{R}$ and $\rho_1(x)=1+\varphi^2(x)$, then the last property is equivalent to $\text{st}_A\text{-}\lim_n\|T_n\varphi^\nu-\varphi^\nu\|_{\rho_1}=0$ for $\nu=0,1,2$.

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MSC:

41A36 Approximation by positive operators
41A25 Rate of convergence, degree of approximation
47B38 Linear operators on function spaces (general)

Keywords:

A-density; A-statistical convergence; sequence of positive linear operators; weight function; weighted space; Korovkin type theorem

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