Let $F$ be an oriented surface of genus $g$, $\text{Diff}(F; \partial)$ the topological group of orientation preserving diffeomorphisms fixing the (possibly empty) boundary. If $g > 1$, the components of $\text{Diff}(F; \partial)$ are contractible and $\text{BDiff}(F; \partial) = B\Gamma'(F)$ where $\Gamma'(F) = \pi_0(\text{Diff}(F; \partial))$ is the mapping class group. The Mumford conjecture is that the rational cohomology of $B\Gamma'(F)$ in dimensions low compared to $g$ is a polynomial algebra over classes $\kappa_i$ in $H_{2i}(B\Gamma(F); \mathbb{Q})$. Let $F_{g,1+1}$ be a surface with two boundary components and mapping class group $\Gamma_{g,1+1}$. $F_{g,1+1}$ is included in $F_{g+1,1+1}$ by gluing a torus with two boundary components onto $F_{g,1+1}$. One obtains maps $B\Gamma^+_g \to B\Gamma^+_{g+1}$ and $B\Gamma^+_{1+1} \to B\Gamma^+_{1+1}$ where $+$ is Quillen’s plus construction. The homotopy direct limit of these maps is $B\Gamma^+_{\infty}$. Mumford’s conjecture takes the form $H^*(B\Gamma(F); \mathbb{Q}) = \mathbb{Q}[\kappa_1, \kappa_2, \ldots]$. It was previously shown that $Z \times B\Gamma^+_\infty$ has an infinite loop space structure. There is an infinite loop map $\alpha_\infty : Z \times B\Gamma^+_\infty \to \Omega^\infty CP^\infty_2$ which is conjectured to be a homotopy equivalence. $\Omega^\infty CP^\infty_2$ is the colim $\Omega^{2s+2}Th(-L_s)$ and $Th(-L_s)$ is the Thom space of the complementary $\mathbb{C}^s$ bundle of the canonical line bundle over $\mathbb{C}P^s$. This is an extension of Mumford’s conjecture. Pursuant to that conjecture the authors obtain the following splitting theorem. For $p$ an odd prime, let the “Adams” splitting of $\Sigma^\infty(CP^\infty_2)^p$ be $E_0, E_1, \ldots, E_{p-2}$. It is shown that there is an infinite loop space $W_p$ such that $(Z \times B\Gamma^+_\infty)^p \simeq \Omega^\infty(E_0) \times \Omega^\infty(E_1) \cdots \times \Omega^\infty(E_{p-3}) \times W_p$.

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