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Two fixed-point theorems for mappings satisfying a general contractive condition of integral type. (English) Zbl 1052.47052

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The author proves fixed point theorems for mappings satisfying a general contractive inequality of integral type.

Theorem: Let (X,d) be a complete metric space, $k \in [0,1)$, let $f : X \to X$ and $\varphi : \mathbb{R}^+ \to \mathbb{R}^+$ be a Lebesgue-integrable mapping which is summable, nonnegative and such that

$$\int_0^{\varepsilon} \varphi(t)\,dt > 0 \quad \text{for each } \varepsilon > 0.$$

Case A. Let

$$m(x,y) = \max\{d(x,y),\, d(x,fx),\, d(y,fy),\, \tfrac{1}{2}[d(x,fy)+d(y,fx)]\}.$$

If f is a mapping such that for each $x, y \in X$

$$\int_0^{d(fx,fy)} \varphi(t)\,dt \le k \int_0^{m(x,y)} \varphi(t)\,dt,$$

then f has a unique fixed point $z \in X$, and for each $x \in X$, $\lim_{n\to\infty} f^n x = z$.

Case B. Let

$$M(x,y) = \max\{d(x,y),\, d(x,fx),\, d(y,fy),\, d(x,fy),\, d(y,fx)\}.$$

If f is a mapping such that for each $x, y \in X$

$$\int_0^{d(fx,fy)} \varphi(t)\,dt \le k \int_0^{M(x,y)} \varphi(t)\,dt$$

and for some $x \in X$ the orbit is bounded, then f has a unique fixed point $z \in X$.

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