Juhász, István; Shelah, Saharon; Soukup, Lajos; Szentmiklóssy, Zoltán
Cardinal sequences and Cohen real extensions. (English) Zbl 1052.54004

Let a scattered space $X$ be decomposed into the Cantor-Bendixson levels $I_\alpha(X), \alpha < ht(X)$, where the height $ht(X)$ is the minimal ordinal number $\alpha$ such that $I_\alpha(X) = \emptyset$. The set $CS(X) := \{ |I_\alpha(X)| : \alpha < ht(X) \}$ is said to be the cardinal sequence of $X$ and the width of $X$ is defined as $wd(X) = \sup \{ |I_\alpha(X)| : \alpha < ht(X) \}$. Strengthening a result of W. Just [Algebra Univ. 20, 135–142 (1985; Zbl 0571.03022)], obtained using a model of $\text{CH}$, the authors prove that by adding any number of Cohen reals to the ground model (without any assumption), in the resulting generic extension there are no locally compact scattered spaces of height $\omega_2$ and width $\omega$. It is also shown in $\text{ZFC}$ that the classes of regular scattered spaces and zero-dimensional scattered spaces have the same cardinal sequences.

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MSC:
54A25 Cardinality properties (cardinal functions and inequalities, discrete subsets)
06E05 Structure theory of Boolean algebras
54G12 Scattered spaces
03E35 Consistency and independence results

Keywords:
locally compact scattered space; superatomic Boolean algebra; Cohen reals; cardinal sequences; zero-dimensional

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