Ali, Majid M.

The Ohm type properties for multiplication ideals. (English) Zbl 1054.13500

Beitr. Algebra Geom. 37, No. 2, 399-414 (1996).

Let R be a commutative ring with identity. Given a nonempty collection $\{I_\lambda\}_{\lambda\in\Lambda}$ of ideals with $I=\sum_\lambda I_\lambda$, it is of interest to know when

$(*k)$ $(\sum_\lambda I_\lambda)^k=\sum_\lambda I_\lambda^k$, or

$(**k)$ $(\bigcap_\lambda I_\lambda)^k=\bigcap_\lambda I_\lambda^k$

holds. For example, it is well known that if R is a Prüfer domain then $(*k)$ holds for all natural numbers k while $(**k)$ holds for all natural numbers k when Λ is finite, but need not hold in general. The purpose of this paper is to determine more general conditions under which $(*k)$ or $(**k)$ holds. The author first proves that if $I=\sum_\lambda I_\lambda$ is a multiplication ideal (i.e., each ideal contained in I is a multiple of I), then $(*k)$ holds for each $k\geq 1$.

He then claims that if $I=I_1+\dots+I_n$ is a multiplication ideal, then $(I_1\cap\dots\cap I_n)^k=I_1^k\cap\dots\cap I_n^k$. This is not correct as may be seen by taking $R=K[X^2,X^3]$, K a field, $I_1=X^2R$, $I_2=X^4R$, $I_3=X^5R$ and $k=2$. (The proof, while correct for the case $n=2$, seems to assume that a subsum of $n-1$ ideals from $\{I_1,\dots,I_n\}$ is a multiplication ideal.) This error aside, the paper does contain a number of other correct results involving multiplication ideals.

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