Nathanson, Melvyn B.
A functional equation arising from multiplication of quantum integers. (English)

Starting from the observation that the numbers \([n]_q = 1 + q + q^2 + \cdots + q^{n-1}\) (which the author calls quantum integers) satisfy \([mn]_q = [m]_q[n]_q\), the author seeks to describe those sequences \(f_n\) of polynomials that satisfy \(f_{mn}(q) = f_m(q)f_n(q^m)\). This goal is not completely achieved. It is shown that under the restriction that \(\deg(f_n) = n - 1\) and \(f_n(0) = 1\) for all \(n\) this is the only solution. Without those restrictions other solutions do exist. The author describes certain classes of solutions and ways to generate new solutions from known ones. Also a result is proved that reduces the general question to the description of those solutions where every nonvanishing \(f_n\) has constant term 1.

Reviewer: Imre Z. Ruzsa (Budapest)

MSC:

11B65 Binomial coefficients; factorials; \(q\)-identities
39B05 General theory of functional equations and inequalities

Keywords:
quantum integer; quantum polynomial; functional equation; additive bases

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References:


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