Xin, Yuanlong
Minimal submanifolds and related topics. (English) ❘Zbl 1055.53047❘
Nankai Tracts in Mathematics 8. River Edge, NJ: World Scientific (ISBN 981-238-687-4). viii, 262 p. (2003).

This book presents the Douglas-Rado solution to the famous Plateau problem, but the main emphasis is on the Bernstein problem and its new developments. In chapter 1, important properties of minimal submanifolds in Euclidean space and in the sphere are proved. Chapter 2 deals mainly with minimal surfaces in $\mathbb{R}^3$. The notion of the Gauss map and the Weierstrass representation are introduced in detail for the proofs of theorems, which generalize the outstanding theorem of Bernstein.

Weierstrass type representations play a major role in chapter 3, too. Theorems are proved, which refer to surfaces in $\mathbb{R}^3$ with prescribed mean curvature as well as to surfaces in hyperbolic space H^3 with constant curvature 1. Chapter 4, presents the Douglas-Rado solution to the Plateau problem. In chapter 5, the author begins with an outline of Kähler geometry, in order to construct a lot of area minimizing submanifolds of higher codimension in Kähler manifolds or in calibrated manifolds. In chapter 6, it is proved that the minimal graphs are not only stable minimal hypersurfaces, but they are also minimizing hypersurfaces. This property leads to two natural generalizations of Bernstein's theorem. One is for higher-dimensional graphs and the other is for the case of stable minimal surfaces in $\mathbb{R}^3$.

Chapter 7 begins with the geometry of Grassmannian manifolds and continues with the study of the Gauss maps whose images lie in Grassmannian manifolds, in order for some results, which refer to Bernstein type theorems for higher codimension, to be proved. The last chapter, 8 is devoted to space-like submanifolds in pseudo-Euclidean space. The chapter mainly studies the external submanifolds, which have vanishing mean curvature, and submanifolds with parallel mean curvature. Among other things some Bernstein type theorems are obtained.

This is an important book and is suitable for students who are familiar with the fundamentals of Riemannian geometry, as well as for specialists.

Reviewer: T. Koufogiorgos (Ioannina)

MSC:

53C42	Differential geometry of immersions (minimal, prescribed curvature, tight, etc.)
53-02	Research exposition (monographs, survey articles) pertaining to differential geometry
52C40	Oriented matroids in discrete geometry
53A10	Minimal surfaces in differential geometry, surfaces with prescribed mean curvature
53C55	Global differential geometry of Hermitian and Kählerian manifolds

Cited in **2** Reviews
Cited in **56** Documents

Keywords:

minimal submanifolds; Plateau problem; Bernstein problem; Weierstrass representation; Kähler geometry; calibrated manifolds; pseudo-Euclidean space