He, Wansheng; Li, Wantong; Yan, Xinxue
Global attractivity of the difference equation $x_{n+1} = \alpha + (x_{n-k}/x_n)$. (English) Zbl 1056.39021

The authors consider the following rational recursive sequence: $x_{n+1} = \alpha + (x_{n-k}/x_n)$, $n = 0, 1, 2, \ldots$ where $\alpha \in (-\infty, 1)$ is a real number, $k \geq 1$ is an integer, and the initial conditions $x_{-k}, \ldots, x_0$ are arbitrary real numbers. They investigate the periodic character, invariant intervals and the global attractivity of all negative solutions. They prove the following

**Theorem:** (1) (i) If $\alpha < -3$, then the equilibrium $\bar{x}$ of Eq. (*) is locally asymptotically stable.

(ii) If $\alpha \in [-2, -1) \cup (-1, 0)$, then the equilibrium $\bar{x}$ of Eq. (*) is unstable.

(iii) If $\alpha \in [-3, -2) \cup [0, 1)$, then the equilibrium $\bar{x}$ of Eq. (*) is a saddle point.

(2) Eq. (*) has no negative solution with prime period two for $\alpha \neq 1$.

(3) Let $\tau > 0$ be an arbitrary positive real number. Assume that $\alpha \in (-\infty, -(\tau + 5)]$, and the initial values $x_{-k}, \ldots, x_0 \in [\alpha - \tau, \alpha + 2]$. Then the interval $[\alpha - \tau, \alpha + 2] + \tau, \alpha$ is an invariant interval of Eq. (*).

(4) Assume $\alpha \in (-\infty, -(\tau + 5)]$. Then the unique negative equilibrium $\bar{x}$ of Eq. (*) is a global attractor $\bar{x}$ with a basin $S = [\alpha - \tau, \alpha + 2] + \tau, \alpha$.


Reviewer: Akira Tsutsumi (Suita)

**MSC:**
- 39A12 Discrete version of topics in analysis
- 39A10 Additive difference equations

**Keywords:**
- difference equation; global attractivity; stability; period two solution; asymptotics; negative solutions; invariant interval; basin; asymptotic stability

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