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Operator means and spectral integration of Fourier multipliers. (English) Zbl 1056.42008

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A bounded operator U on $L^p(\mu)$ ($U \in B(L^p(\mu))$) is said to be trigonometrically well-bounded if U has a spectral representation $U = \int_{0-}^{2\pi} e^{it}\, dE(t)$, where $E : \mathbb{R} \to B(L^p(\mu))$ is an idempotent-valued function called (after suitable normalization) the spectral decomposition of U. Now, if $\Psi \in BV(\mathbb{T})$, the integral $\int_{0-}^{2\pi} \Psi(e^{it}) dE(t)$ exists strongly as a Riemann-Stieltjes integral and the mapping

$$\Psi \longrightarrow \Psi(U) = \Psi(1)E(0) + \int_{0-}^{2\pi} \Psi(e^{it}) dE(t) \tag{1}$$

is a norm continuous representation of the Banach algebra $BV(\mathbb{T})$ in $B(L^p(\mu))$. The goal of this paper, which is a continuation of a previous paper of the authors [Ill. J. Math. 43, No. 3, 500–519 (1999; Zbl 0930.42004)] is to find sufficient conditions on $T \in B(L^p(\mu))$ to ensure that T will be trigonometrically well-bounded with a spectral decomposition that extends the spectral integration in (1) from $BV(\mathbb{T})$ to the Marcinkiewicz q-class of Fourier multipliers $M_q(\mathbb{T})$. The main result of this paper asserts that if T is a bounded, invertible and separation-preserving mapping on $L^p(\mu)$ such that T is mean$_2$-bounded, that is, for every $f \in L^p(\mu)$ and every $N \in \mathbb{N}$,

$$\frac{1}{(2N+1)^{1/2}} \left\| \left\{ \sum_{k=-N}^{N} |T^k f|^2 \right\}^{1/2} \right\|_{L^p(\mu)} \leq A ||f||_{L^p(\mu)}$$

the result follows.

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MSC:

42A45 Multipliers in one variable harmonic analysis
42B15 Multipliers for harmonic analysis in several variables
46E30 Spaces of measurable functions (L^p-spaces, Orlicz spaces, Köthe function spaces, Lorentz spaces, rearrangement invariant spaces, ideal spaces, etc.)
47B40 Spectral operators, decomposable operators, well-bounded operators, etc.

Keywords:

Lebesgue spaces; spectral decomposition; Fourier multiplier; separation preserving operator; mean boundedness