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David Hilbert's lectures on the foundations of geometry, 1891–1902. (English) Zbl 1057.01009
David Hilbert's Lectures on the Foundations of Mathematics and Physics, 1891–1933 1. Berlin: Springer (ISBN 3-540-64373-7/hbk). xxviii, 661 p. (2004).

This hefty volume contains previously unpublished lecture notes of David Hilbert on the foundations of geometry (with other volumes on foundations of mathematics and physics forthcoming). The notes were important to Hilbert since he frequently cites them in his subsequent lectures and are annotated in his own hand. The work is published after 15 years of support and involvement by numerous individuals and institutions enabling opening a working office near archives in Göttingen. Hilbert first lectured in Königsberg, then in Göttingen and undoubtedly influenced the 20th century mathematics, not only by his publications, but through his lectures and interactions with the 'Hilbert circle.' Lecture notes provide a fuller picture of his evolving thought as well as different interconnections between various research interests. Lecture notes reveal his preoccupation with deepening the foundations of mathematics and natural science; he once called it: 'Tieferlegung der Fundamente' Only a fragment of surviving manuscripts is published.

The sources were mainly 1. The library of the Mathematisches Institut der Georg-August Universität in Göttingen, 2. The Handschriftenabteilung (Manuscript Division) of the Niedersächsische Staats- und Universitätsbibliothek, Göttingen. There are three types of manuscripts: a) Those prepared by Hilbert himself for his own use, b) Polished protocols done by his (graduate) students, assistants and collaborators, worked out after the lectures and c) In class lecture notes. Some Hilbert manuscripts were written by his wife Käthe. The criteria for selection of texts were first that they came from the foundational texts and to provide the reader a glimpse into how Hilbert developed his ideas. Overlaps of the material were to be avoided. Texts exist for virtually every of his lectures on geometry. Each chapter has an English title and is preceded by an introductory note discussing the significance the contents, the historical and scientific background. The documents themselves are in the original German language. Some editorial changes of the texts are reported in the footnotes, but many inessential ones are not.

The volume contains: 1. Hilbert's own notes for a (slightly less than 3/4 of the) course on projective geometry held in Königsberg in the Summer semester of 1891. Part of the content involves points and lines at infinity, eight 'fundamental laws of intuition' in essence four pairs of dual incidence axioms, a proof of the two-dimensional Duality Principle, Desargues' theorem in planar and spatial version with their converses, harmonic points, lines and planes, notions of perspective and projectivity and their relationships, fundamental theorem of projective geometry, second order configurations, such as Brianchon's Theorem and its dual, Pascal's (Pappus') Theorem (so important in Hilbert's later works)...

2. His notes from 1893 to 1894 for a course on the foundations of geometry, given in Königsberg in the summer semester of 1894, announced originally for 1893. This is his axiomatic geometry to the fullest and its importance in Hilbert's developments of foundations of science cannot be overestimated,

3. Hilbert's notes of (just over a half of) the two 'Holiday Courses' (*Ferienkurse*) for school teachers, given in Göttingen in the winter semester of 1898/1899. Among other things Hilbert argues here that school teachers should familiarize themselves with research mathematics as well as that the latter practitioners should pay attention to school mathematics.

4. Two sets of notes for a course on the elements of Euclidean geometry, held at Göttingen in the Winter semester of 1898/1899, one in Hilbert's hand, another *Ausarbeitung* by his student Hans von Schaper. There are also Hilbert's notes on the origins of the Euclidean Geometry here.

5. Complete republication of the original edition of the so-called *Festschrift*, the first edition of Hilbert's celebrated work *Grundlagen der Geometrie*, published in 1899, and Hilbert's important additions written for the first French translation of 1902, retained in the first English translation of 1902. The editors' commentary seems to be most extensive here.

6. Notes for a course on the foundations of geometry delivered in Göttingen during the Summer semester of 1902. Here, there is for instance, axiomatization of Bolyai-Lobatchevsky geometry that uses only the familiar plane incidence, order and congruence axioms, and a suitable non-Euclidean axiom for parallels;

continuity axioms are avoided. To be sure, Hilbert uses a number of resources for his own lectures, including Gauss, Klein, then Theodor Reye's textbook *Geometrie der Lage, 1886*, Pasch's *Vorlesungen über die neuere Geometrie, 1882...*

At the end of the book there is a table of Hilbert's courses he held throughout his career, with dates and manuscript whereabouts. The bibliography is rather useful and it contains all the important personalities related to geometry or this work in particular. There is no index.

Undoubtedly an important book individual and university libraries should have. One can look forward in expectation to the remaining volumes of Hilbert's lectures on foundations of mathematics and physics.

Reviewer: [Radoslav M. Dimitrić \(Galveston\)](#)

MSC:

[01A75](#) Collected or selected works; reprintings or translations of classics
[51-03](#) History of geometry

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| Cited in 5 Reviews Cited in 15 Documents |
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Biographic references:

[Hilbert, David](#)