Popescu, Calin

On the homology of free Lie algebras. (English) Zbl 1059.17503

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Let H, $\mathbb{L}$, and U denote the homology functor, the free Lie algebra functor, and the universal enveloping algebra functor, respectively. Let K be a field of characteristic zero. *D. G. Quillen* has proved [Ann. Math. (2) 90, 205-295 (1969; Zbl 0191.53702)] that if V is a differential graded K-vector space, then the natural homomorphism $\mathbb{L}H(V) \to H\mathbb{L}(V)$ is an isomorphism of graded K-Lie algebras, and if L is a differential graded K-Lie algebra, then the natural homomorphism $UH(L) \to HU(L)$ is an isomorphism of graded cocommutative K-Hopf algebras. There results are no more valid if we replace K by a field of non-zero characteristic or by a ring of characteristic zero containing $1/2$. The author has discovered that factoring out the torsion in homology enables to generalize the above Quillen's results. Let F denote the free part. He shows that if R is a principal ideal domain of characteristic zero containing $1/2$ and V is a connected differential non-negatively graded R-free module of finite type, then the natural homomorphisms $\mathbb{L}FH(V) \to FH\mathbb{L}(V)$ of graded R-Lie algebras, and $UFH\mathbb{L}(V) \to FHU\mathbb{L}(V)$ of graded cocommutative R-Hopf algebras, are both isomorphisms. In the end the author presents several interesting examples showing how these results can be used within the framework of the Quillen's models.

Reviewer: Jiří Vanžura (Brno)

MSC:

17B55 Homological methods in Lie (super)algebras
17B01 Identities, free Lie (super)algebras
17B35 Universal enveloping (super)algebras

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