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Existence of solutions for a class of hyperbolic systems of conservation laws in several space dimensions. (English) Zbl 1061.35048

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The authors study the Cauchy problem for the system of conservation laws

$$\partial_t u + \sum_{\alpha=1}^{m} \partial_{x_\alpha} f_\alpha(|u|)u = 0, \quad u = (u_1, \dots, u_n) \in \mathbb{R}^n,$$

with initial condition $u(0,\cdot) = \bar{u}$. As was recently shown by Bressan, this problem can be ill-posed for general initial data $\bar{u} \in L^\infty$. The authors consider the case $\bar{u} \in BV_{loc}(\mathbb{R}^n)$ and prove existence of the entropy solution. The radial part $\rho = |u|$ is required to be a Kruzhkov's entropy solution for the scalar conservation law

$$\partial_t \rho + \sum_{\alpha=1}^{m} \partial_{x_\alpha} f_\alpha(\rho)\rho = 0$$

with corresponding initial data $\rho(0,\cdot) = |\bar{u}|$, while the angular part $\theta = u/\rho$ must satisfy the linear transport equation

$$\partial_t \rho\theta + \sum_{\alpha=1}^{m} \partial_{x_\alpha} f_\alpha(\rho)\rho\theta = 0$$

with coefficients $(\rho, f_\alpha(\rho)\rho) \in BV_{loc}$. The authors use the theory recently developed by the first author [Invent. Math. 158, No. 2, 227–260 (2004; Zbl 1075.35087)] to prove existence of weak solution θ of the transport equation, which satisfies the key condition $|\theta| = 1$, thus proving existence of the entropy solution $u = \rho\theta$ of the original problem. The paper also contains some remarks about conditions of uniqueness and stability of entropy solutions.

Reviewer: Evgeniy Panov (Novgorod)

MSC:

35L65 Hyperbolic conservation laws
35L45 Initial value problems for first-order hyperbolic systems
35F10 Initial value problems for linear first-order PDEs

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