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A singularly perturbed linear eigenvalue problem in C^1 domains. (English) Zbl 1061.35061

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For any $\gamma > 0$, set

$$\Lambda(\gamma) = \sup_{u\in H^1(\Omega)\setminus\{0\}} \frac{\gamma\int_{\partial\Omega} u^2 - \int_\Omega |\nabla u|^2}{\int_\Omega u^2},$$

where Ω is a bounded domain in $\mathbb{R}^n$ with boundary $\partial\Omega \in C^1$. The supremum is attained by some positive function $u_\gamma \in H^1(\Omega)$, which is a weak solution of

$$\Delta u = \Lambda(\gamma)u \quad \text{in } \Omega, \qquad \frac{\partial u}{\partial \nu} = \gamma u \quad \text{on } \partial\Omega,$$

where ν is the outward unit normal vector on $\partial\Omega$. The goal of this paper is to understand the asymptotic behavior of $\Lambda(\gamma)$ as $\gamma \to \infty$. Since $\Lambda(\gamma) \to \infty$ when $\gamma \to \infty$, this problem can be viewed as a singularly perturbed linear eigenvalue problem. The following theorems are proved.

Theorem 1.

$$\lim_{\gamma\to\infty} \frac{\Lambda(\gamma)}{\gamma^2} = 1$$

holds for any bounded C^1 domain.

Theorem 2. If $a > 1$, then

$$\Delta u = au \quad \text{in } \mathbb{R}^n_+, \qquad \frac{\partial u}{\partial x_n} = -u \quad \text{on } \partial\mathbb{R}^n_+,$$

has no bounded nontrivial solution. Here a is the limit of $\frac{\Lambda(\gamma)}{\gamma^2}$ (subject to a subsequence) as $\gamma \to \infty$.

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MSC:

35P15 Estimates of eigenvalues in context of PDEs
35B40 Asymptotic behavior of solutions to PDEs
35B25 Singular perturbations in context of PDEs
35J25 Boundary value problems for second-order elliptic equations

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