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Learning classes of approximations to non-recursive functions. (English) Zbl 1061.68085

Summary: L. Blum and M. Blum [Inf. Control 28, 125–155 (1975; Zbl 0375.02028)] showed that a class $B$ of suitable recursive approximations to the halting problem $K$ is reliably EX-learnable but left it open whether or not $B$ is in NUM. By showing $B$ to be not in NUM we resolve this old problem. Moreover, variants of this problem obtained by approximating any given recursively enumerable set $A$ instead of the halting problem $K$ are studied. All corresponding function classes $U(A)$ are still EX-inferable but may fail to be reliably EX-learnable, for example if $A$ is non-high and hypersimple. Blum and Blum (loc. cit.) considered only approximations to $K$ defined by monotone complexity functions. We prove this condition to be necessary for making learnability independent of the underlying complexity measure. The class $B$ of all recursive approximations to $K$ generated by all total complexity functions is shown to be not even behaviorally correct learnable for a class of natural complexity measures. On the other hand, there are complexity measures such that $B$ is EX-learnable. A similar result is obtained for all classes $\tilde{U}(A)$. For natural complexity measures, $B$ is shown to be not robustly learnable, but again there are complexity measures such that $B$ and, more generally, every class $U(A)$ is robustly EX-learnable. This result extends the criticism of S. Jain et al. [J. Comput. Syst. Sci. 62, No. 1, 178–212 (2001; Zbl 0992.68176)], since the classes defined by artificial complexity measures turn out to be robustly learnable while those defined by natural complexity measures are not robustly learnable.

MSC:
68Q32 Computational learning theory
03D25 Recursively (computably) enumerable sets and degrees
68T05 Learning and adaptive systems in artificial intelligence

Full Text: DOI

References:
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