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**Minimal projections in Banach spaces. Problems of existence and uniqueness and their application.** (English) [Zbl 1062.46500](#)

*Lecture Notes in Mathematics* 1449. Berlin etc.: Springer-Verlag (ISBN 3-540-53197-1). viii, 168 p. (1990).

The present manuscript is an extension of lectures offered by Odyniec during the academic year 1984–1985. Chapters I, II and IV in the current manuscript are a translation of extensions of those lectures. Chapter III in the manuscript is new work dealing with Kolmogorov-type criteria for minimal projections.

The basic problem is the following: Given a Banach space  $X$  and a subspace  $M$ , does there exist a projection  $P: X \rightarrow M$  such that  $P$  has minimum norm of all projections of  $X$  onto  $M$ ? The manuscript deals mainly with the uniqueness and strong uniqueness problems for minimal projections.

The introduction gives a road map for the manuscript. The translation is somewhat uneven but with a little perseverance the reader is rewarded immensely.

Chapter I discusses the uniqueness problems of minimal projections on arbitrary Banach spaces. The authors give excellent geometric intuition and provide worthwhile figures to aid the reader in understanding the basic concepts. The reader is led gently to the fact that an understanding of minimal projections onto subspaces of codimension one is essential as one attempts to move to the discussion of minimal projections onto subspaces of higher codimensions.

Chapter II presents a clever relationship between the uniqueness problem for minimal projection onto subspaces of codimension one and a carefully proposed mathematical programming problem. Solutions for  $l_1$  and  $l_\infty^n$  are given as well as results for the spaces  $l_1$  and  $c_0$ .

In Chapter III, the space  $C(T)$  is discussed and Kolmogorov type characterizations are given that enable one to consider the problem of strongly unique best approximation projections.

The last chapter studies isometries of a Banach space onto itself and characterizes Hilbert spaces within the class of uniformly smooth strictly smooth Banach spaces, using the theory of minimal projections.

The majority of the material in the manuscript covers developments over the ten years 1975–1985. There are 193 references with the most recent reference being in 1988. This monograph should be required reading for any investigator who wishes to obtain a detailed overview of the theory of minimal projections.

Reviewer: Joseph M. Lambert (MR 92a:41021)

**MSC:**

- 46-02 Research exposition (monographs, survey articles) pertaining to functional analysis
- 46B20 Geometry and structure of normed linear spaces
- 41A65 Abstract approximation theory (approximation in normed linear spaces and other abstract spaces)
- 46N10 Applications of functional analysis in optimization, convex analysis, mathematical programming, economics
- 49J40 Variational inequalities
- 90C48 Programming in abstract spaces

Cited in **4** Reviews  
Cited in **23** Documents

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minimal projection; projection constant

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