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Imaginary quadratic fields k with $\mathrm{Cl}_2(k) \simeq (2, 2^m)$ and rank $\mathrm{Cl}_2(k^1) = 2$. (English)

Zbl 1063.11038

Pac. J. Math. 198, No. 1, 15-31 (2001).

The imaginary quadratic fields k such that both k and its Hilbert 2-class field have 2-class groups of rank 2 are characterized in this article. For a number field F, let $h_2(F)$, $\mathrm{Cl}_2(F)$ and F^1 denote its 2-class number, 2-class group and Hilbert 2-class field respectively. Also, the pair $(2, 2m)$ denotes the group $\mathbb{Z}_2 \times \mathbb{Z}_{2^m}$.

The main theorem is: Let k be an imaginary quadratic field with $\mathrm{Cl}_2(k) \cong (2, 2^m)$. Then rank $\mathrm{Cl}_2(k^1) = 2$ if and only if $k = \mathbb{Q}(\sqrt{-p_1p_2p_3})$, where p_1, p_2 and p_3 are primes satisfying $p_1 \not\equiv 3 \not\equiv p_2 \pmod 4$ and $p_3 \equiv 3 \pmod 4$, $(\frac{p_1}{p_2}) = -1$, $(\frac{p_1}{p_3}) = (\frac{p_2}{p_3}) = 1$ and $h_2(K) = 2$, where K is a non-normal quartic subfield of one of the two unramified cyclic quartic extensions of k such that $\mathbb{Q}(\sqrt{p_1p_2}) \subset K$. For example, the fields $\mathbb{Q}(\sqrt{-d})$ for $d = 310, 406, 598, 1443$ and 1615 all satisfy these conditions.

Reviewer: Charles Parry (Blacksburg)

MSC:

11R29 Class numbers, class groups, discriminants
11R11 Quadratic extensions

Cited in **6** Documents

Keywords:

Hilbert 2-class field

Full Text: DOI

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