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Morita theory for corings and cleft entwining structures. (English) Zbl 1064.16037

The authors study Morita contexts and generalize and unify results of Chase and Sweedler, of Doi and of Cohen, Fischman and Montgomery. Let $A$ and $R$ be rings with $i: A \to R$ a ring morphism, and $\chi: R \to A$ a map satisfying conditions necessary and sufficient for $A$ to be a right module over $R$ via $a \mapsto r = \chi(ar)$. For any right $R$-module $M$, $M^R = \{m \in M : m \cdot r = m\chi(r)\}$. Then there is a Morita context $(B, R, A, Q, \tau, \mu)$ where $B = A^R$, $Q = R^R$, $\tau: A \otimes_R Q \to B$ is given by $\tau(a \otimes_R q) = \chi(qa)$, and $\mu: Q \otimes_B A \to R$ is given by $\mu(q \otimes_B a) = qa$. Indeed this context is the Morita context associated to the right $R$-module $A$ in the usual way. If, furthermore, $R/A$ is Frobenius, then $(B, R, A, A, \tau, \mu)$ is a Morita context as above with $A \cong Q$ as $(R, B)$ modules.

Next the authors consider the situation where $A$ is a ring and $(C, x)$ is an $A$-coring with $x$ a fixed grouplike element. In other words, $C$ is a coalgebra in the monoidal category of $(A, A)$-bimodules; $x$ grouplike means that $\Delta_C(x) = x \otimes_C x$ and $\varepsilon(x) = 1$. Then $\chi: R = \ast C \to A$ defined by $\chi(f) = f(x)$ satisfies the assumptions above, and a Morita context can be constructed. Also there is a Morita context connecting $R = \ast C$ and $B' = A^C$, namely $(B', R, A, Q', \tau', \mu')$ with a morphism between the two contexts. If $\tau'$ is surjective or if $C$ is finitely generated and projective as a left $A$-module, the two contexts coincide.

In the next section, the authors look at the particular situation where the coring $C = A \otimes C$, i.e. $C$ arises from an entwining structure $(A, C, \psi)$.

In the fifth section, the authors consider factorization structures and the Morita context of Cohen, Fischman and Montgomery.

In the final section, cleft factorization structures are defined and discussed.

Reviewer: Margaret Beattie (Sackville)

MSC:

16W30 Hopf algebras (associative rings and algebras) (MSC2000)
16D90 Module categories in associative algebras
16S40 Smash products of general Hopf actions
16L60 Quasi-Frobenius rings

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Morita contexts; corings; entwining structures; cleft comodule algebras; grouplike elements; coalgebras; monoidal categories; smash products

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