Graphs considered in this paper are finite, undirected, and possibly with parallel edges or loops. The maximum genus \( \gamma_M(G) \) of a graph \( G = (V, E) \) is the largest integer \( k \) such that \( G \) can be 2-cell embedded into an orientable surface \( S \) of genus \( k \). Euler’s formula implies that \( \gamma_M(G) \leq \lfloor \beta(G)/2 \rfloor \), where \( \beta(G) = |E(G)| - |V(G)| + 1 \). The graph \( G \) is said to be upper embeddable if \( \gamma_M(G) = \lfloor \beta(G)/2 \rfloor \). Let \( T \) be a spanning tree of the connected graph \( G \). Let \( \xi(G, T) \) denote the number of components of \( G \setminus E(T) \) that have odd number of edges. The Betti deficiency \( \xi(G) \) is defined to be \( \min_T \xi(G, T) \), where the minimum is taken over all spanning trees \( T \) of \( G \). It is known that \( \gamma_M(G) = (\beta(G) - \xi(G))/2 \) and \( G \) is upper embeddable if and only if \( \xi(G) \leq 1 \).

The main theorem proved in this paper reads as follows. Let \( G \) be a connected graph satisfying \( \xi(G) = k \geq 2 \). For any spanning tree \( T \) of \( G \), there exist \( k \) vertex-disjoint connected induced subgraphs \( H_1, \ldots, H_k \) such that, for every \( 1 \leq i \leq k \), (i) \( \beta(H_i) \) is an odd number, and (ii) \( T \cap H_i \) is connected and the set \( E(H_i, G) \) consisting of edges that have one end in \( H_i \) and the other end not in \( H_i \) is included in \( E(T) \). Some consequences of the maximum genus of a graph are derived from this main result.

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