Howe, Everett W.

On the non-existence of certain curves of genus two. (English) Zbl 1067.11035

Let $q$ be a power of an odd prime and $f(x) = f_q(x) := x^4 + (2 - 2q)x + q^2$. For $q \leq 61$ D. Maisner and E. Nart [Exp. Math. 11, 321–337 (2002)] noticed that there is no curve of genus 2 over $F_q$ whose characteristic polynomial is $f(x)$. The objective of this paper is to show that such statement holds for any $q$. The prove is based on a counting argument. First the number of principally polarized abelian surfaces with characteristic polynomial $f(x)$ is counted. It turns out that this number is equal to the number of geometrically split principally polarized abelian surfaces whose characteristic polynomial is $f(x)$. In particular, $f(x)$ cannot be the characteristic polynomial of a Jacobian. It turns out that for an abelian surface with characteristic polynomial $f(x)$, $(\text{End}A) \otimes \mathbb{Q} = K$ where $K = \mathbb{Q}(\sqrt{-2}, \sqrt{-D})$, $2q - 1 = F^2D$ with $D$ squarefree. Hence the approach of proof of the main result in this paper is based on several properties of the subfields of $K$.

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MSC:

11G20 Curves over finite and local fields
11G10 Abelian varieties of dimension $> 1$
11R65 Class groups and Picard groups of orders
14G15 Finite ground fields in algebraic geometry
14H25 Arithmetic ground fields for curves

Keywords:

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