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Computing Fitting ideals of Iwasawa modules. (English) Zbl 1067.11067
Math. Z. 246, No. 4, 733-767 (2004).

Let K/F be a finite Abelian Galois extension of number fields, with F totally real and K CM, and denote, for a given odd prime number p , by K_∞, F_∞ the cyclotomic \mathbb{Z}_p -extensions of K, F , respectively. The paper is concerned with computing the (initial) Fitting ideals of the minus parts of the Iwasawa modules X_{std}, X_{du} over the Iwasawa algebra $\mathbb{Z}_l[[G_{K_\infty/F}]]$. Here, X_{std} is the ‘standard’ Iwasawa module, i.e., the projective limit of the p -parts of the class groups in the cyclotomic tower over K , and X_{du} is a certain ‘dual’ module, namely the Galois group of the maximal Abelian p -extension of K_∞ over HK_∞ , where H is the p -class field of the minimal field K_n in the tower K_∞/K so that all p -adic places become totally ramified in K_∞/K_n .

The first result describing the Fitting ideal of a closely related module Y_S (compare e.g. [*J. Ritter* and *A. Weiss*, Mem. Am. Math. Soc. 748 (2002; [Zbl 1002.11082](#)))] has Brumer-Stickelberger type consequences on the size of the annihilator of X_{du}^- and of the p -part of the minus class group of K . The next sections are devoted to the actual calculation of the Fitting ideals of X_{du}^- and X_{std}^- (outside the Teichmüller character when $\zeta_p \in K$, but for X_{std}^- also at the Teichmüller character of the p -regular part of $G_{K/F}$); it is assumed here that the μ -invariant of X_{std}^- vanishes. The overlap between these and recent results of Kurihara is discussed. Under appropriate assumptions the validity of the p -part of the Brumer-Stark conjecture is a consequence of the shown computations (the main assumption is the nontrivial zeroes condition).

Reviewer: [Jürgen Ritter \(Augsburg\)](#)

MSC:

- [11R23](#) Iwasawa theory
- [11R42](#) Zeta functions and L -functions of number fields
- [12G10](#) Cohomological dimension of fields
- [13D02](#) Syzygies, resolutions, complexes and commutative rings

Cited in **7** Reviews
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