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Cauchy problem for fractional diffusion equations. (English) Zbl 1068.35037
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Equations of the form

$$(D_t^{(\alpha)}u)(t, x) - Bu(t, x) = f(t, x), \quad t \in [0, \tau], \quad 0 < \alpha < 1, \quad x \in \mathbb{R}^n$$

where

$$(D_t^{(\alpha)}u)(t, x) = \frac{1}{\Gamma(1-\alpha)} \left[\frac{\partial}{\partial t} \int_0^t (t-\zeta)^{-\alpha} u(\zeta, x) d\zeta - t^{-\alpha} u(0, x) \right]$$

$$B = \sum_{k,j=1}^n a_{kj}(x) \frac{\partial^2}{\partial x_k \partial x_j} + \sum_{j=1}^n b_j(x) \frac{\partial}{\partial x_j} + c(x)$$

are considered here. The fundamental solution is studied via a Green matrix. The arguments of the Green matrix are expressed in terms of Fox's H -functions. Estimates of the elements of the Green matrix are also presented.

Reviewer: [Chaman Lal Koul \(Jaipur\)](#)

MSC:

35K15 Initial value problems for second-order parabolic equations
26A33 Fractional derivatives and integrals

Cited in **178** Documents

Keywords:

[Fox's \$H\$ -function](#); [fundamental solution](#)

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