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From constant mean curvature hypersurfaces to the gradient theory of phase transitions.
(English) Zbl 1070.58014

Let \((M, g)\) be a compact Riemannian manifold and let \(W : \mathbb{R} \to \mathbb{R}\) be an non-negative smooth function vanishing at the nondegenerate minimum points 1 and -1. For any \(\varepsilon > 0\), it is considered the energy functional \(E_\varepsilon\) on \(H_0^1(\Omega)\) given by

\[
E_\varepsilon(u) = \varepsilon^2 \int_M (|\nabla u|^2_g + W(u)) dv_g.
\]

The authors prove that, if \(N\) is an admissible nondegenerate minimal hypersurface of \(M\), then there exists a sequence of critical points \(u_\varepsilon\) of \(E_\varepsilon\) whose nodal sets converge to \(N\) as \(\varepsilon \to 0\). Then it is shown that a similar result holds if \(N\) is an admissible volume-nondegenerate constant mean curvature hypersurface of \(M\) and the functionals \(E_\varepsilon\) are subject to a suitable volume constraint.

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MSC:
58E12 Variational problems concerning minimal surfaces (problems in two independent variables)
35B25 Singular perturbations in context of PDEs
35J20 Variational methods for second-order elliptic equations
53A10 Minimal surfaces in differential geometry, surfaces with prescribed mean curvature

Keywords:
critical point; Riemannian manifold; hypersurface; curvature

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