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A fixpoint theory for non-monotonic parallelism. (English) Zbl 1070.68081

Summary: This paper studies parallel recursion. The trace specification language used in this paper incorporates sequentially, nondeterminism, reactivity (including infinite traces), three forms of parallelism (including conjunctive, fair-interleaving and synchronous parallelism) and general recursion. In order to use Tarski’s theorem to determine the fixpoints of recursions, we need to identify a well-founded partial order. Several orders are considered, including a new order called the lexical order, which tends to simulate the execution of a recursion in a similar manner as the Egli-Milner order. A theorem of this paper shows that no appropriate order exists for the language. Tarski’s theorem alone is not enough to determine the fixpoints of parallel recursions. Instead of using Tarski’s theorem directly, we reason about the fixpoints of terminating and nonterminating behaviours separately. Such reasoning is supported by the laws of a new composition called partition.

We propose a fixpoint technique called the partitioned fixpoint, which is the least fixpoint of the nonterminating behaviours after the terminating behaviours reach their greatest fixpoint. The surprising result is that although a recursion may not be lexical-order monotonic, it must have the partitioned fixpoint, which is equal to the least lexical-order fixpoint. Since the partitioned fixpoint is well defined in any complete lattice, the results are applicable to various semantic models. Existing fixpoint techniques simply become special cases of the partitioned fixpoint. For example, an Egli-Milner-monotonic recursion has its least Egli-Milner fixpoint, which can be shown to be the same as the partitioned fixpoint. The new technique is more general than the least Egli-Milner fixpoint in that the partitioned fixpoint can be determined even when a recursion is not Egli-Milner monotonic. Examples of non-monotonic recursions are studied. Their partitioned fixpoints are shown to be consistent with our intuition.

MSC:
68Q55 Semantics in the theory of computing
68Q10 Modes of computation (nondeterministic, parallel, interactive, probabilistic, etc.)

Keywords:
Recursion; Reactive programs; Parallelism; Semantics; Partitioned fixpoint

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Z

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References: