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Analogue p-adique du théorème de Turrittin et le théorème de la monodromie p-adique.

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The aim of this paper is to prove the basic structure theorem about p-adic differential equations. Another proof of the same theorem is available in [*Y. André*, "Filtrations de type Hasse-Arf et monodromie p-adique." Invent. Math. 148, No. 2, 285–317 (2002; Zbl 1081.12003)]. Even if both proofs are based on the same properties (mainly on the integrality of the p-adic irregularity) they are very distinct. The one given here follows rather faithfully (with many extra difficulties) the classical proof of Turrittin theorem for differential equations over a formal power series field. It is then as effective as possible, even if it uses several tools, for instance the decomposition theorem, which are far from being effective.

Let K be a discretely valued complete field of characteristic 0 with residue field k of characteristic p and let $\mathcal{R} = \mathcal{R}_{K,x}$ be the so called Robba ring (namely the ring of power series $\sum_{n\in\mathbb{Z}} a_n\, x^n$, $(a_n \in K)$ that converges in some annulus $1-\varepsilon < |x| < 1$). Any finite separable extension of $k((x))$ can be written $k'((t))$, for some finite extension k' of k and with x in $t\,k'[t]$. It can then be lifted in an extension $\mathcal{R}_{K',t}$ of $\mathcal{R}$. The p-adic analog of Turrittin theorem says that for any $\mathcal{R}$-differential module (namely $\mathcal{R}[d/dx]$-module, free of finite rank as $\mathcal{R}$-module) $\mathcal{M}$ which is endowed with a Frobenius structure (namely such that, for $\varphi(x) = x^p$, $\mathcal{M}$ and $\varphi^*(\mathcal{M}) := \mathcal{R} \underset{\varphi}{\nwarrow} \mathcal{R}$

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MSC:

12H25	p-adic differential equations
14F30	p-adic cohomology, crystalline cohomology
11F80	Galois representations

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