Inequalities for quantum relative entropy. (English) Zbl 1076.15019

Linear Algebra Appl. 401, 159-172 (2005).

Given positive semi-definite matrices $A, B \in M_n(C)$, the authors prove the following log-majorization:

$$A^{(1+q)/2} B^q A^{(1+q)/2} \prec (\log) A^{1/2} B^p A^{1/2},$$

where $0 < q \leq p$. This inequality is then used to reprove a result from T. Ando and F. Hiai [Linear Algebra Appl. 197–198, 113–131 (1994; Zbl 0793.15011)], and this result is then used to prove several inequalities regarding $\alpha$-power means. The rest of the paper is devoted to proving a generalized thermodynamic inequality and its equivalence with the Peierls-Bogoliubov inequality and others.

Reviewer: Martín Argerami (Regina)

MSC:

15A45  Miscellaneous inequalities involving matrices
15A90  Applications of matrix theory to physics (MSC2000)
47A63  Linear operator inequalities
82B10  Quantum equilibrium statistical mechanics (general)

Keywords:
relative entropy; logarithmic trace inequalities; Peierls-Bogoliubov inequality; thermodynamic inequality

Full Text: DOI Link

References:

[8] Furuta, T., $(A\geq B\geq 0)$ ensures $(B^{r}A^{-\frac{1}{r}}B^{\frac{1}{r}}) \leq (\frac{1}{q}A\geq (B^{p}A^{\frac{1}{p}}B^{\frac{1}{p}})^{q/p})A^{1/2}$ for $(r\geq 0, p\geq 0, q\geq 1, \frac{1}{q}+(1+2r)q=\frac{p}{q}+(p+2r)$, Proc. Amer. Math. Soc., 101, 85-88 (1987) · Zbl 0721.47023


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