Gluskin, E.

On the multivariable version of Ball's slicing cube theorem. (English) Zbl 1077.52004

Milman, V. D. (ed.) et al., Geometric aspects of functional analysis. Papers from the Israel seminar (GAFA) 2002–2003. Berlin: Springer (ISBN 3-540-22360-6/pbk). Lecture Notes in Mathematics 1850, 117-121 (2004).

The author proves the following far-reaching generalization of *K. Ball*'s cube slicing theorem ["Volumes of sections of cubes and related problems", Lect. Notes Math. 1376, 251–260 (1989; Zbl 0674.46008)]. Let V_j $(j = 1, 2, \ldots, m)$ be compact subsets of $\mathbb{R}^n$ (n-dimensional Euclidean space) each of Lebesgue measure one. Let H_C be the subspace of $\mathbb{R}^{nm}$ defined by $H_C := \{(x_1, x_2, \ldots, x_m) \in \mathbb{R}^{nm} : \sum_{j=1}^m c_{ij}x_j = 0$ for $i = 1, 2, \ldots, k\}$ where $C = (c_{ij})$ is an arbitrary $k \times m$ real matrix and the x_j's are in $\mathbb{R}^n$. Then $\text{meas}_H\{H_C \cap (V_1 \times V_2 \times \cdots V_m)\} \leq \exp(kn/2)$. The proof uses Ball's argument and *F. Barthe*'s multidimensional version of the Brascamp-Lieb inequality [Invent. Math. 134, 335–361 (1998; Zbl 0901.26010)]. Evidently one can translate the V_j's and so the result applies to flats H_C as well as subspaces.

For the entire collection see [Zbl 1052.46001].

Reviewer: A. C. Thompson (Halifax)

MSC:

52A20 Convex sets in n dimensions (including convex hypersurfaces)
52A38 Length, area, volume and convex sets (aspects of convex geometry)

Keywords:

slices of cube; slices of product sets; subspace of product space; Brascamp-Lieb inequality