Ramlau, Ronny; **Teschke, Gerd**

Tikhonov replacement functionals for iteratively solving nonlinear operator equations. (English) Zbl 1078.47030

Inverse Probl. 21, No. 5, 1571-1592 (2005).

The paper is concerned with an ill-posed nonlinear equation $F(x) = y$, where $F : X \to Y$ is a twice continuously differentiable operator between Hilbert spaces X and Y. Let $y^\delta \in Y$ be an available approximation to y with $\|y^\delta - y\| \leq \delta$. The authors suggest a two-level Tikhonov-type regularization method $x_{k+1} = \operatorname{argmin}_{x\in X} \widetilde{J}_\alpha(x, x_k)$, $k = 0, 1, \dots$, $\alpha \to 0$, where $\widetilde{J}_\alpha(x, a) = J_\alpha(x) + C\|x - a\|^2 - \|F(x) - F(a)\|^2$, $C, \alpha > 0$, and $J_\alpha(x) = \|y^\delta - F(x)\|^2 + \alpha\|x - \overline{x}\|^2$ is the classical Tikhonov functional. Let the derivative $F'(x)$ be Lipschitz continuous on X with a constant L. It is shown that the modified functional $\widetilde{J}_\alpha(x, x_k)$ is strictly convex and that x_{k+1} can be obtained by the fixed point iteration, provided that $L\sqrt{J_\alpha(a)} < C + \alpha$. Moreover, if $\|F(x) - F(\widetilde{x}) - F'(\widetilde{x})(x - \widetilde{x})\| \leq \|F(x) - F(\widetilde{x})\|$ $\forall x, \widetilde{x} \in X$ and the solution x^* to the original equation satisfies $x^* - \overline{x} \in R(F'^*(x^*))$, then the method with an appropriate stopping rule $\alpha = \alpha(\delta)$ generates approximations converging to x^* as $\delta \to 0$.

Reviewer: Mikhail Yu. Kokurin (Yoshkar-Ola)

MSC:

47J06 Nonlinear ill-posed problems

47J25 Iterative procedures involving nonlinear operators

Keywords:

two-level Tikhonov-type regularization; fixed point iteration

Software:

TIGRA

Full Text: DOI