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Some remarks on $\mathfrak{g}$-variant Fedosov star products and quantum momentum mappings.

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P. Xu [Commun. Math. Phys. 197, No. 1, 167–197 (1998; Zbl 0939.37048)] studied the notion of quantum momentum mapping for $\mathfrak{g}$-invariant star products and raised the question whether the existence of a quantum momentum mapping follows from the existence of a classical momentum mapping. This question is answered in the negative in the paper under review, and the method used here improves results of *K. Hamachi* [Rev. Math. Phys. 14, No. 6, 601–621 (2002; Zbl 1040.53097)] and *S. Gutt* [Star products and group actions, Bayrischzell Workshop, April 26–29 (2002)].

The first part of the present paper includes a brief review of *B. V. Fedosov*'s construction of star products on symplectic manifolds. Additionally, one describes the derivations of the corresponding star products, and then one compares the Fedosov derivations obtained from different data. In Section 3 one obtains necessary and sufficient conditions in order for the Lie derivative with respect to a symplectic vector field to be a derivation of a Fedosov star product (Theorem 3.8). This result is then used to investigate the invariance of the star product with respect to a Lie algebra action. One further obtains criteria for existence of quantum Hamiltonians, eventually leading to the negative answer to P. Xu's aforementioned question.

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MSC:

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53D20 Momentum maps; symplectic reduction

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