Bertolini, Massimo; Darmon, Henri; Iovita, Adrian; Spiess, Michael
Teitelbaum's exceptional zero conjecture in the anticyclotomic setting. (English)
Zbl 1079.11036
Am. J. Math. 124, No. 2, 411-449 (2002).

Let ϕ be an eigenform of even weight $k \geq 2$ on $\Gamma_0(N)$. In "On p-adic analogues of the conjectures of Birch and Swinnerton-Dyer" [Invent. Math. 84, 1–48 (1986; Zbl 0699.14028)] *B. Mazur, J. Tate* and *J. Teitelbaum* formulated a p-adic variant of the conjecture of Birch and Swinnerton-Dyer – the case $k = 2$ – for the value of the p-adic L-function $L_p(\phi, s)$ attached to the cyclotomic $\mathbb{Z}_p$-extension of $\mathbb{Q}$ at $s = 1$. In the special case that p divides N exactly and that the p-th coefficient of ϕ equals 1, the p-adic L-function $L_p(\phi, s)$ vanishes at $s = 1$. In this situation the conjectures of Mazur, Tate and Teitelbaum imply the following relationship between the derivative $L_p'(\phi, 1)$ and the special value $L(\phi, 1)$ of the classical L-function at 1:

$$L_p'(\phi, 1) = \mathcal{L}(\phi) \cdot L(\phi, 1)/\Omega,$$

where Ω is a real period and $\mathcal{L}(\phi)$, the so-called $\mathcal{L}$-invariant, is defined using p-adic uniformization. This relation was proved by *R. Greenberg* and *G. Stevens* [Invent. Math. 111, 407–447 (1993; Zbl 0778.11034)].

For arbitrary modular forms of even weight k *J. Teitelbaum* [Invent. Math. 101, 395–410 (1990; Zbl 0731.11065)] suggested a definition of the $\mathcal{L}$-invariant, which should give the analog of the Greenberg-Stevens result in the form

$$L_p'(\phi, k/2) = \mathcal{L}(\phi) \cdot L(\phi, k/2)/\Omega.$$

In this paper the authors prove an analogue of Teitelbaum's conjecture, in which the cyclotomic $\mathbb{Z}_p$-extension of $\mathbb{Q}$ is replaced by the anti-cyclotomic $\mathbb{Z}_p$-extension of an imaginary quadratic number field K, in which the prime p splits. This generalizes results of *M. Bertolini* and *H. Darmon* [Duke Math. J. 98, 305–334 (1999; Zbl 1037.11045)] and emphasizes the role of p-adic integration. The authors also consider the case that the prime p is inert in K, which leads to an interpretation of $L_p'(\phi, k/2)$ in terms of p-adic Coleman integrals and generalizes one of the main results of *M. Bertolini* and *H. Darmon* [Invent. Math. 131, 453–491 (1998; Zbl 0899.11029)] to even weights $k \geq 2$.

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MSC:

11G40 L-functions of varieties over global fields; Birch-Swinnerton-Dyer conjecture
11F33 Congruences for modular and p-adic modular forms
11F67 Special values of automorphic L-series, periods of automorphic forms, cohomology, modular symbols
11G18 Arithmetic aspects of modular and Shimura varieties

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