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Equivalence and embedding problems for CR-structures of any codimension. (English)


The authors discuss in this paper a new approach to the general problem of establishing the CR equivalence of CR manifolds of arbitrary codimension. Recall that a CR manifold $M$, with analytic tangent space $HM$, is of finite type [in the sense of T. Bloom and I. Graham, Invent. Math. 40, 217–243 (1977; Zbl 0339.32003)] if the smooth sections of $HM$ are generators of the Lie algebra of smooth vector fields on $M$. Denote by $J$ the partial complex structure on $HM$ and set $T^{0,1}M = \{ X + iJX \mid X \in HM \}$. Let $T^{*0}M$ and $T^{*1,0}M$ be the subbundles of the complexified cotangent bundle $CT^*M$ consisting of the forms that annihilate $HM$ and $T^{0,1}M$, respectively. Then $T^{*0}M \subset T^{*1,0}M$ and the Lie derivative $L_Z$, for $Z \in C^\infty(M, T^{0,1}M)$, maps $C^\infty(M, T^{*0}M)$ into itself. Then $M$ is finitely nondegenerate at $p \in M$ if the span of $L_{Z_1} \circ \cdots \circ L_{Z_m} (\theta)(p)$, for $Z_1, \ldots, Z_m \in C^\infty(M, T^{0,1}M)$, and $\theta \in C^\infty(M, T^{*0})$ is $T^{*1,0}M$ [this notion was introduced by C. K. Han, Invent. Math. 73, 51–69 (1983; Zbl 0517.32007)].

For CR manifolds $M, M'$, which are at the same time of finite type and finitely nondegenerate, the Authors show that there exist a positive integer $r$ and a smooth map $\Phi : G^r(M, M') \to G^{r+1}(M, M')$ such that every smooth CR-diffeomorphism $f : M \to M'$ satisfies a complete differential system of the form $j^r_{1x} = \Phi(j^r_1(x))$. Here $G^h(M, M')$ is the set of $h$-jets of smooth diffeomorphisms $f : M \to M'$ and $j^r_1(f)$ is the $h$-jet of $f$ at $x \in M$. Moreover, if $M$ is connected, a CR diffeomorphism is completely determined by its $r$-jet at any point of $M$.

The authors also consider, and reduce to the study of complete differential systems, the problem of the CR embedding of $M$ into some other CR manifold $M'$ and give also interesting examples and other applications.

The main feature and the novelty of this paper in the literature of CR-equivalence is that the authors do not make any real analyticity assumption for the CR manifolds involved.

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MSC:

32V25 Extension of functions and other analytic objects from CR manifolds
53A40 Other special differential geometries
32V35 Finite-type conditions on CR manifolds

Keywords:

CR structure; Equivalence problem; Nondegeneracy conditions; Complete systems; Jet parametrization

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