German, O. N.
Sails and norm minima of lattices. (English. Russian original) Zbl 1084.11035
Sb. Math. 196, No. 3, 337-365 (2005); translation from Mat. Sb. 196, No. 3, 31-60 (2005).

The main result of this paper is the following Theorem: The norm minimum of an n-dimensional lattice $\Lambda \subset \mathbb{R}^n$ is non-zero if and only if there is a uniform bound on the determinants of the faces of each of the 2^n sails generated by the lattice Λ and the standard cone $\mathcal{C}_0 : \{(t_1, \dots, t_n \,|\, t_i > 0\}$. Here the *norm minimum* of Λ is defined to be $\inf_{(x_1,\dots,x_n)\in\Lambda\setminus\{0\}} |x_1 \dots x_n|$; a *sail* is the boundary of the convex hull of the intersection of a cone $\mathcal{C}$ with $\Lambda \setminus \{0\}$; the sails generated by Λ and $\mathcal{C}_0$ arise from letting $\mathcal{C}$ be generated by the various choices for $(\pm e_1, \dots, \pm e_n)$, where the e_i form the canonical basis of $\mathbb{R}^n$; each sail is a (generalized) $n-1$ polytope, faces are as usual; the *determinant* of a face is its normalized volume (so that the determinant of a simplicial face is the determinant of the matrix with entries the components of the vertices).

Reviewer: Thomas Schmidt (Corvallis)

MSC:

11H50	Minima of forms
11J70	Continued fractions and generalizations
11H06	Lattices and convex bodies (number-theoretic aspects)
52C07	Lattices and convex bodies in n dimensions (aspects of discrete geometry)

Keywords:

multidimensional continued fractions

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