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Liapunov spectra for infinite chains of nonlinear oscillators. (English) Zbl 1084.37500

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Summary: We argue that the spectrum of Lyapunov exponents for long chains of nonlinear oscillators, at large energy per mode, may be well approximated by the Lyapunov exponents of products of independent random matrices. If, in addition, statistical mechanics applies to the system, the elements of these random matrices have a distribution which may be calculated from the potential and the energy alone. Under a certain isotropy hypothesis (which is not always satisfied), we argue that the Lyapunov exponents of these random matrix products can be obtained from the density of states of a typical random matrix. This construction uses an integral equation first derived by Newman. We then derive and discuss a method to compute the spectrum of a typical random matrix. Putting the pieces together, we see that the Lyapunov spectrum can be computed from the potential between the oscillators.

MSC:

37A30 Ergodic theorems, spectral theory, Markov operators
60K35 Interacting random processes; statistical mechanics type models; percolation theory
70K99 Nonlinear dynamics in mechanics
82B05 Classical equilibrium statistical mechanics (general)

Keywords:

Lyapunov exponents; random matrices; coupled oscillators

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