Reichel, Lothar; Ye, Qiang

Breakdown-free GMRES for singular systems. (English) Zbl 1086.65030

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The generalized minimal residual (GMRES) method is based on the Arnoldi process, which may suffer from benign or hard breakdown. First the authors investigate in which space the solution of the system (or the least squares solution in the case of an inconsistent system) $Ax = b$ can be found. If $\mathcal{K}_{n-1}$ is the Krylov subspace at hard breakdown, then the solution is in $\mathcal{K}_{n-1} + \mathcal{N}(A^p)$, which requires the inclusion of an eigenvector of A^p corresponding to the eigenvalue 0, to extend the Krylov sequence. This is the basis of their algorithm to overcome hard breakdown.

Another breakdown-free variant of the range restricted version was given by *D. Calvetti, B. Lewis*, and *L. Reichel* [Linear Algebra Appl. 316, No. 1–3, 157–169 (2000; Zbl 0963.65042)]. The present method is also related to, but different from the method proposed by *Q. Ye* [Math. Comput. 62, No. 205, 179–207 (1994; Zbl 0796.65046)].

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MSC:

65F10	Iterative numerical methods for linear systems
65F20	Numerical solutions to overdetermined systems, pseudoinverses

Keywords:

GMRES; iterative methods; Krylov subspace method; singular matrix; linear system; Arnoldi method; least squares solution; inconsistent system; generalized minimal residual method

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