

Cauty, Robert

A fixed-point theorem for acyclic multivalued functions. (Un théorème de point fixe pour les fonctions multivoques acycliques.) (French) [Zbl 1088.55001](#)

Kadets, Vladimir (ed.) et al., Functional analysis and its applications. Proceedings of the international conference, dedicated to the 110th anniversary of Stefan Banach, Lviv National University, Lviv, Ukraine, May 28–31, 2002. Amsterdam: Elsevier (ISBN 0-444-51373-6/hbk). North-Holland Mathematics Studies 197, 71-80 (2004).

Suppose that $F : X \multimap X$ is an upper semi-continuous (usc) multivalued function such that, for each $x \in X$, the set $F(x)$ is closed, nonempty and acyclic, that is, it has trivial reduced rational cohomology. It follows from a classical result of *D. Montgomery* and *S. Eilenberg* [*Am. J. Math.* 68, 214–222 (1946; [Zbl 0060.40203](#))] that if X is a compact absolute retract (AR), then F has a fixed point, that is, $x \in F(x)$ for some $x \in X$.

This result extends to noncompact ARs by requiring that F be compact, that is, each $F(x)$ is compact and the union of the $F(x)$ for all $x \in X$ is contained in a compact subset of X . If X is an AR, then it is uniformly contractible, that is, there exists a map $\lambda : X \times X \times I \rightarrow X$ such that $\lambda(x, y, 0) = x$, $\lambda(x, y, 1) = y$ for all $x, y \in X$ and $\lambda(x, x, t) = x$ for all $x \in X$ and $t \in I$. Convex subsets of linear topological spaces and contractible topological groups also have the property of being uniformly contractible.

The purpose of the paper is to prove that every compact, usc and acyclic-valued multivalued function from a separable uniformly contractible space X to itself has a fixed point. In the case that X is also compact and metrizable, the result is reduced to the AR case. The further extension to all separable uniformly contractible spaces is obtained by means of projective limits.

For the entire collection see [[Zbl 1076.46001](#)].

Reviewer: [Robert F. Brown \(Los Angeles\)](#)

MSC:

- [55M20](#) Fixed points and coincidences in algebraic topology
- [54H25](#) Fixed-point and coincidence theorems (topological aspects)
- [54C60](#) Set-valued maps in general topology

Cited in 1 Document

Keywords:

[fixed points](#); [acyclic multivalued functions](#)