Let $L^2_\alpha(K)$, $\alpha > 0$, be the space of square integrable functions on $K := [0, \infty) \times \mathbb{R}$ with respect to the measure $dm_\alpha(y, s) : (\pi \Gamma(\alpha + 1))^{-1} y^{2\alpha+1} dy ds$ and let $H^\nu_\alpha(K)$, $\nu \in \mathbb{R}$, be a Hilbert space of some functions, belonging to the $L^2_\alpha(K)$. The Laguerre-type Weierstrass transform $L^r f, r > 0, of f \in H^\nu_\alpha(K)$ is considered. It is proved that for any $g \in L^2_\alpha(K)$, and for any $\mu > 0$, the infimum $\inf_{f \in H^\nu_\alpha(K)} \{ \mu \| f \|_{H^\nu_\alpha(K)} + \| g - L^r f \|_{L^2_\alpha(K)} \}$ is attained by a unique function $f_{\mu, \nu, g}$. The construction and approximate properties of these functions are established.

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MSC:
- 41A50 Best approximation, Chebyshev systems
- 42B10 Fourier and Fourier-Stieltjes transforms and other transforms of Fourier type
- 44A20 Integral transforms of special functions
- 46E22 Hilbert spaces with reproducing kernels (= (proper) functional Hilbert spaces, including de Branges-Rovnyak and other structured spaces)

Keywords:
best approximation; Laguerre-type Weierstrass transform; Hilbert space; Fourier-Laguerre transform

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