Ungar, Abraham A.

Analytic hyperbolic geometry. Mathematical foundations and applications. (English)

Zbl 1089.51003
Hackensack, NJ: World Scientific (ISBN 981-256-457-8/hbk). xviii, 463 p. (2005).

This book represents an exposition of the author's single-handed creation, over the past 17 years, of an algebraic language in which both hyperbolic geometry and special relativity find an aesthetically pleasing formulation, very much like Euclidean geometry and Newtonian mechanics find them in the language of vector spaces. The aim is thus to provide not just an analytic geometry to enable algebraic computations to answer questions formulated in hyperbolic geometry, as such analytic hyperbolic geometries existed for all the models of hyperbolic geometry, but rather to provide a counterpart to the inner product vector space model of Euclidean geometry. The resulting counterparts allow for formulas that look very much like their Euclidean counterpart, with the difference that the vectors are not elements of a vector space, but rather of a gyrovector space, that the operations of addition, multiplication with scalars, and inner product, are all "gyro", and do not satisfy the familiar commutative, associative rules, but rather "gyro"-variations of these, creating an elaborate "gyrolanguage", in which all terms familiar from the Euclidean setting get their gyro-counterpart. Besides an abstract theory of gyrovector spaces, the author concentrates on three special cases: Möbius, Einstein, and PV (proper velocity) gyrovector spaces, all of which can serve as models for hyperbolic geometry. To exemplify the similarity with the Euclidean setting, the author proves an apparent Pythagorean theorem for hyperbolic geometry, as well as the fact that the medians of a triangle meet in a point. The author greatly emphasizes the fact that, in this algebraic setting, the original Einstein velocity addition finds a natural home, in which it becomes gyrocommutative, and would have perhaps not been abandoned by the advent of Minkowski's space-time reformulation of special relativity, had the gyrolanguage existed at the time. The connection with special relativity is emphasized throughout.

For a different way of looking at the connection between hyperbolic geometry and special relativity, the reader may wish to consult *W. Benz* [Classical geometries in modern contexts. Geometry of real inner product spaces (Birkhäuser, Basel) (2005; Zbl 1084.51001)].

Reviewer: Victor V. Pambuccian (Phoenix)

MSC:

51M10	Hyperbolic and elliptic geometries (general) and generalizations
51-02	Research exposition (monographs, survey articles) pertaining to geometry
51P05	Classical or axiomatic geometry and physics
83A05	Special relativity
20N05	Loops, quasigroups
51N25	Analytic geometry with other transformation groups

Keywords:

gyrogroups