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Alternative stable homotopy classification of $BG_p^\wedge$.


In [Topology 34, 633–649 (1995; Zbl 0835.55011)], J. Martino and S. Priddy gave necessary and sufficient conditions for the $p$-completed classifying spaces $BG_p^\wedge$ and $BG_p'^\wedge$ of two finite groups $G$ and $G'$ to be stably homotopy equivalent. In the same paper they also provided an example of two finite groups whose $p$-completed classifying spaces are stably homotopy equivalent but not (unstably) homotopy equivalent. B. Oliver on the other hand recently has shown in [Mem. Am. Math. Soc. 848, 102 p. (2006; Zbl 1095.55008) and Math. Proc. Camb. Philos. Soc. 137, No. 2, 321–347 (2004; Zbl 1077.55006)] that $BG_p^\wedge$ and $BG_p'^\wedge$ are homotopy equivalent provided there is a fusion preserving group isomorphism $S \to S'$ for a $p$-Sylow subgroup $S$ of $G$ and a $p$-Sylow subgroup $S'$ of $G'$. Here an isomorphism $\varphi : S \to S'$ is called fusion preserving if for any two subgroups $P$ and $Q$ of $S$ a group isomorphism $\alpha : P \to Q$ is induced by conjugation in $G$ if and only if the corresponding isomorphism $\varphi\alpha\varphi^{-1} : \varphi(P) \to \varphi(Q)$ is induced by a conjugation in $G'$.

Using the Lewis-May-McClure version of the Segal conjecture [L. G. Lewis, J. P. May and J. E. McClure, Current trends in algebraic topology, Semin. London/Ont. 1981, CMS Conf. Proc. 2, No. 2, 165–179 (1982; Zbl 0572.55006)], the author of the present paper shows that the fusion system for a pair $(G, S)$ consisting of a finite group $G$ and a choice of a $p$-Sylow subgroup $S$ can be recovered from the stable category over $\Sigma^\infty BG_p^\wedge$. From this he obtains the main result of the paper which says that $BG_p^\wedge$ and $BG_p'^\wedge$ are homotopy equivalent if and only if there is an isomorphism $S \to S'$ between $p$-Sylow subgroups $S \subset G$ and $S' \subset G'$ and a (stable) homotopy equivalence $\Sigma^\infty BG_p^\wedge \to \Sigma^\infty BG_p'^\wedge$ which make the obvious associated diagram of $p$-completed classifying spaces in the stable homotopy category commutative.

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