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Mixed integer linear programming in process scheduling: modeling, algorithms, and applications. (English) [Zbl 1091.90055](#)

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Summary: This paper reviews the advances of mixed-integer linear programming (MILP) based approaches for the scheduling of chemical processing systems. We focus on the short-term scheduling of general network represented processes. First, the various mathematical models that have been proposed in the literature are classified mainly based on the time representation. Discrete-time and continuous-time models are presented along with their strengths and limitations. Several classes of approaches for improving the computational efficiency in the solution of MILP problems are discussed. Furthermore, a summary of computational experiences and applications is provided. The paper concludes with perspectives on future research directions for MILP based process scheduling technologies.

MSC:

[90C11](#) Mixed integer programming

[90B35](#) Deterministic scheduling theory in operations research

[90C57](#) Polyhedral combinatorics, branch-and-bound, branch-and-cut

[90-02](#) Research exposition (monographs, survey articles) pertaining to operations research and mathematical programming

Cited in **21** Documents

Keywords:

chemical process scheduling; mixed-integer linear programming (MILP); discrete-time model; continuous-time model; branch and bound

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