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Curvature and rank of Teichmüller space. (English) Zbl 1092.32008

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Let S be a surface with genus g and n boundary components, and let $d(S) = 3g - 3 + n$ denote the number of curves in any pants decomposition of S. The Weil-Petersson metric on the Teichmüller space $\mathrm{Teich}(S)$ has many curious properties. It is a Riemannian metric with negative sectional curvature, but its curvatures are not bounded away from zero or negative infinity. It is geodesically convex, but it is not complete.

In this paper the authors show that in spite of exhibiting negative curvature behavior, the Weil-Petersson metric is not coarsely negatively curved except for topologically simple surfaces S. The main theorem answers a question of Bowditch.

Theorem 1.1. Let S be a compact surface of genus g with n boundary components. Then the Weil-Petersson metric on $\mathrm{Teich}(S)$ is Gromov-hyperbolic if and only if $3g - 3 + n \leq 2$. The constant $d(S) = 3g - 3 + n$ is the complex dimension of the Teichmüller space $\mathrm{Teich}(S)$.

To summarize, the overlap of the positive and negative results in this paper give: When $d(S) = 2$, the Weil-Petersson metric on $\mathrm{Teich}(S)$ is Gromov-hyperbolic, yet $\mathrm{Teich}(S)$ admits no $\mathrm{Mod}(S)$-invariant, complete, Riemannian metric with pinched negative curvature.

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MSC:

32G15	Moduli of Riemann surfaces, Teichmüller theory (complex-analytic aspects in several variables)
30F60	Teichmüller theory for Riemann surfaces
53B35	Local differential geometry of Hermitian and Kählerian structures
53B40	Local differential geometry of Finsler spaces and generalizations (areal metrics)
53C60	Global differential geometry of Finsler spaces and generalizations (areal metrics)

Keywords:

Weil-Petersson metric on Teichmüller space; negative sectional curvature; Gromov-hyperbolic metric

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