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A solution procedure for second-order difference equations and its application to electromagnetic-wave diffraction in a wedge-shaped region. (English) Zbl 1092.78008

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Summary: This paper proposes an efficient solution procedure for second-order functional difference equations, and outlines this procedure through investigating electromagnetic-wave diffraction by a canonical structure comprising an impedance wedge and an impedance sheet bisecting the exterior region of the wedge. Applying the Sommerfeld-Malyuzhinets technique to the original boundary-value problem yields a linear system of equations for the two coupled spectral functions. Eliminating one spectral function leads to a second-order difference equation for the other. The chief steps in this work consist of transforming the second-order equation into a simpler one by making use of a generalized Malyuzhinets function $\chi_\Phi(\alpha)$, and in expressing the solution to the latter in an integral form with help of the so-called S-integrals. From this integral expression one immediately obtains a Fredholm equation of the second kind for points on the imaginary axis of the complex plane. Solving this integral equation by means of the well-known quadrature method enables us to calculate the sought-for spectral function inside the basic strip via an interpolation formula and outside it via an analytic extension. The second spectral function is obtained through its dependence upon the first. The uniform asymptotic solution, which is of particular interest in the geometrical theory of diffraction, follows, by evaluating the Sommerfeld integrals in the far field from the exact one. Several examples demonstrate the efficiency and accuracy of the proposed procedure as well as typical behaviour of the far-field solutions for such a canonical problem of diffraction theory.

MSC:

78A45 Diffraction, scattering
39A05 General theory of difference equations

Keywords:

second-order difference equations; S-integrals; generalized Malyuzhinets function χ_Φ; integral equation of the second kind; exact and uniform asymptotic solution; wedge-shaped region

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