Colmez, Pierre

The p-adic Birch and Swinnerton-Dyer's conjecture. (La conjecture de Birch et Swinnerton-Dyer p-adique.) (French) Zbl 1094.11025

Bourbaki seminar. Volume 2002/2003. Exposes 909–923. Paris: Société Mathématique de France (ISBN 2-85629-156-2/pbk). Astérisque 294, 251-319, Exp. No. 919 (2004).

Let E be an elliptic curve over $\mathbb{Q}$ of rank $r(E)$. For a prime number p factorize the Euler factor in p of the L-function $L(E,s)$ as $(1-\alpha_1 p^{-s})(1-\alpha_2 p^{-s})$ and choose (whenever possible) $\alpha \in \{\alpha_1, \alpha_2\}$ verifying $v_p(\alpha) < 1$. Let $L_{p,\alpha}(E,s)$ be the p-adic L-function of E associated to α.

The author explains the proof of the following important theorem of *K. Kato* [Astérisque 295, 117–290 (2004; Zbl 1142.11336)]: The order of the zero of $L_{p,\alpha}(E,s)$ at $s=1$ is $\geq r(E)$ and $\geq r(E)+1$ if $\alpha = 1$. If equality holds then the p-part of the Tate-Shafarevich group of E is finite an the p-adic regulator $R_{p,\alpha}(E)$ is different from zero.

An extensive bibliography of 194 items closes this brilliant survey.

For the entire collection see [Zbl 1052.00010].

Reviewer: Florin Nicolae (Berlin)

MSC:

11G40 L-functions of varieties over global fields; Birch-Swinnerton-Dyer conjecture

11-02 Research exposition (monographs, survey articles) pertaining to number theory

Keywords:

elliptic curve; p-adic L-function; construction of Kato's Euler system; Kato's explicit reciprocity law; relationship between Kato's Euler system and p-adic L-functions of modular forms