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Compressible flow in a half-space with Navier boundary conditions. (English) Zbl 1095.35025

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The following initial-boundary value problem is considered in the half-space $Q = \{(x,t) : x \in \mathbb{R}^3,\, x_3 > 0,\, t > 0\}$

$$\frac{\partial \rho}{\partial t} + \operatorname{div}(\rho v) = 0 \quad \text{in } Q,$$

$$\frac{\partial}{\partial t}(\rho v_i) + \operatorname{div}(\rho v_i v) + \frac{\partial}{\partial x_i} p(\rho) - \mu \Delta v_i - \lambda \operatorname{div}\left(\frac{\partial v}{\partial x_i}\right) = \rho f_i, \quad i = 1,2,3 \quad \text{in } Q,$$

$$v = K(x)\left(\frac{\partial v_1}{\partial x_3}, \frac{\partial v_2}{\partial x_3}, 0\right), \quad x_3 = 0, \quad t > 0,$$

$$(\rho, v)(x,0) = (\rho_0, v_0)(x), \quad x \in \mathbb{R}^3_+.$$

Here the velocity $v(x,t) = (v_1, v_2, v_3)$ and the density $\rho(x,t)$ are unknown functions, $p = p(\rho)$ is the pressure, f is a given external force, λ and μ are viscosity constants, K is a smooth positive function.

The global in time existence of weak solutions is proved if the data of the problem are small in energy norms. Estimates of solutions are obtained.

Reviewer: Il'ya Sh. Mogilevskij (Tver')

MSC:

35Q30	Navier-Stokes equations
76N10	Existence, uniqueness, and regularity theory for compressible fluids and gas dynamics

Keywords:

Navier-Stokes equations; compressible flow; Navier boundary conditions; existence of weak solutions

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