Cavalier, Vincent; **Lehmann, Daniel**

On the Poincaré inequality for one-dimensional foliations. (English) Zbl 1097.32010

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Authors' abstract: Let d be the degree of an algebraic one-dimensional foliation $\mathcal{F}$ on the complex projective space $\mathbb{P}_n$. Let Γ be an algebraic solution of degree δ, and geometrical genus g.

We prove, in particular, the inequality $(d-1)\delta+2-2g \geq \mathcal{B}(\Gamma)$, where $\mathcal{B}(\Gamma)$ denotes the total number of locally irreducible branches through singular points of Γ when Γ has singularities, and $\mathcal{B}(\Gamma)=1$ when Γ is smooth.

Equivalently, when $\Gamma=\bigcap_{\lambda=1}^{n-1} S_\lambda$ is the complete intersection of $n-1$ algebraic hypersurfaces S_λ, we get $(d+n-\sum_{\lambda=1}^{n-1}\delta_\lambda)\delta \geq \mathcal{B}(\Gamma)-\mathcal{E}(\Gamma)$, where δ_λ denotes the degree of S_λ and $\mathcal{E}(\Gamma)=2-2g+(\sum_\lambda \delta_\lambda-(n+1))\delta$ the correction term in the genus formula. These results are also refined when Γ is reducible.

Reviewer: Andrzej Piatkowski (Łódź)

MSC:

32S65	Singularities of holomorphic vector fields and foliations
14M10	Complete intersections
19E20	Relations of K-theory with cohomology theories
32S20	Global theory of complex singularities; cohomological properties

Keywords:

degree of an algebraic foliation; Poincaré inequality; genus formula; GSV-index

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