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Existence of solutions and star-shapedness in Minty variational inequalities. (English)

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Summary: Minty Variational Inequalities (for short, Minty VI) have proved to characterize a kind of equilibrium more qualified than Stampacchia Variational Inequalities (for short, Stampacchia VI). This conclusion leads to argue that, when a Minty VI admits a solution and the operator F admits a primitive f (that is $F = f'$), then f has some regularity property, e.g. convexity or generalized convexity. In this paper we put in terms of the lower Dini directional derivative a problem, referred to as Minty VI(f'_-, K), which can be considered a nonlinear extension of the Minty VI with $F = f'$ (K denotes a subset of $\mathbb{R}^n$). We investigate, in the case that K is star-shaped, the existence of a solution of Minty VI(f'_-, K) and increasing along rays starting at x^* property of (for short, $F \in \mathrm{IAR}(K, x^*)$). We prove that Minty VI(f'_-, K) with a radially lower semicontinuous function fhas a solution x^* ker K if and only if $f \in \mathrm{IAR}(K, x^*)$. Furthermore we investigate, with regard to optimization problems, some properties of increasing along rays functions, which can be considered as extensions of analogous properties holding for convex functions. In particular we show that functions belonging to the class $\mathrm{IAR}(K, x^*)$ enjoy some well-posedness properties.

MSC:

49J40	Variational inequalities
49K40	Sensitivity, stability, well-posedness
47J20	Variational and other types of inequalities involving nonlinear operators (general)

Keywords:

existence of solutions; generalized convexity; Minty variational inequality; star-shaped sets; well-posedness

Full Text: DOI

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