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Planar periodic systems of population dynamics. (English) Zbl 1098.34037

Cañada, A.(ed.) et al., Ordinary differential equations. Vol. II. Amsterdam: Elsevier/North Holland (ISBN 0-444-52027-9/hbk). Handbook of Differential Equations, 359-460 (2005).

This chapter analyzes the existence, attractiveness and multiplicity of the nonnegative T-periodic solution of the planar periodic system

$$u'(t) = \lambda\ell(t)u(t) - a(t)u^2(t) - b(t)u(t)v(t),$$

$$v'(t) = \mu m(t)v(t) - d(t)v^2(t) - c(t)u(t)v(t),$$

where $\ell > 0, m > 0, a > 0, d > 0$ are smooth T-periodic functions. The authors do not impose any sign restriction on the coupling coefficient function $b(t), c(t)$. The above system includes those of Lotka-Volterra type and a more general class of models simulating symbiotic interactions within global competitive environments. The chapter contains eight sections. In Sections 1 and 2, an introduction and some basic preliminaries are presented for the subsequent mathematical analysis. Section 3 ascertains the linearized stability character of the semi-trivial positive solutions of the system. Section 4 analyzes the minimal complexity of the components of the (λ, μ)-plane determined by the curves of neutral stability of the semi-trivial states. Section 5 gives an abstract unilateral global bifurcation result for the system. Section 6 considers the symbiotic prototype model ($b < 0$ and $c < 0$) and uses the theory of monotone periodic systems to show that the set of coexistence states linking the surfaces of the semi-trivial states along their respective curves of neutral stability. Other interesting results are also presented in Section 6. Section 7 adapts the mathematical analysis of Section 6 to the competing model, which possesses a quasi-cooperative structure. In Section 8, the author briefly discusses some results related to predator-prey models.

For the entire collection see [Zbl 1074.34003].

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MSC:

34C60 Qualitative investigation and simulation of ordinary differential equation models
92D25 Population dynamics (general)
34C25 Periodic solutions to ordinary differential equations
34C23 Bifurcation theory for ordinary differential equations

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Lotka-Volterra systems; symbiosis in competitive systems; coexistence states