Hirosawa, Fumihiki

Loss of regularity for second order hyperbolic equations with singular coefficients. (English)

Zbl 1098.35115
Osaka J. Math. 42, No. 4, 767-790 (2005).

The author is interested in the backward Cauchy problem

$$u_{tt} - \lambda(t)^2 b(t)^2 \Delta u = 0, \quad u(T,x) = u_0(x), \quad u_t(T,x) = u_1(x)$$

in the strip $[0,T] \times \mathbb{R}^n$. The function $\lambda = \lambda(t)$ describes the decreasing behaviour with a degeneracy at $t = 0$, the nonnegative function $b = b(t)$ describes the oscillating behaviour of the coefficient.

The author explains the relation between the interplay of both parts and the loss of regularity of the solution. In the case $0 \le b_0 \le b(t)$ the quantities (for a suitable $\kappa_1 \ge 0$)

$$\sup_{t\in(0,T]} \left(\frac{\Lambda(t)}{\lambda(t)(\ln \Lambda(t)^{-1})^{\kappa_1}} |b'(t)| \right), \quad \sup_{t\in(0,T]} \left(\left(\frac{\Lambda(t)}{\lambda(t)(\ln \Lambda(t)^{-1})^{\kappa_1}} \right)^2 |b''(t)| \right)$$

have an essential influence on the loss of regularity.

Finally, the case that $b = b(t)$ has a countable number of zeros is discussed. The optimality of the results is shown by using Floquet theory.

Reviewer: Michael Reissig (Freiberg)

MSC:

35L80 Degenerate hyperbolic equations
35L15 Initial value problems for second-order hyperbolic equations

Keywords:

weakly hyperbolic equations; oscillations; Floquet theory; backward Cauchy problem