

Ren, Xiumin**On exponential sums over primes and application in Waring-Goldbach problem.** (English)

Zbl 1100.11025

Sci. China, Ser. A 48, No. 6, 785-797 (2005).

Author's abstract: We prove the following estimate on exponential sums over primes: Let $k \geq 1$, $\beta_k = 1/2 + \log k / \log 2$, $x \geq 2$ and $\alpha = a/q + \lambda$ subject to $(a, q) = 1$, $1 \leq a \leq q$ and $\lambda \in \mathbb{R}$. Then

$$\sum_{x < m \leq 2x} \Lambda(m) e(\alpha m^k) \ll (d(q))^{\beta_k} (\log x)^c \left(x^{1/2} \sqrt{q(1 + |\lambda|x^k)} + x^{4/5} + \frac{x}{\sqrt{q(1 + |\lambda|x^k)}} \right).$$

As an application, we prove that with at most $O(n^{7/8+\varepsilon})$ exceptions, all positive integers up to N satisfying some necessary congruence conditions are the sum of three squares of primes. This result is as strong as what has previously been established under the generalized Riemann hypothesis.

Reviewer: Jürgen Hinz (Marburg)

MSC:

11L20 Sums over primes

11P05 Waring's problem and variants

11P32 Goldbach-type theorems; other additive questions involving primes

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