De Angelis, Valerio; Beslin, Scott J.
Rings permitting polynomial interpolation. (English) Zbl 1100.16016

A (not necessarily commutative) ring $R$ is said to have property IP ("interpolating polynomial" property) if for any finite subset $\{(x_1, y_1), (x_2, y_2), \ldots, (x_n, y_n)\}$ of $R \times R$ there exist a nonnegative integer $m$ and elements $a_0 \in R, a_i \in R$ or $a_i$ being an integer, for $i = 1, \ldots, m$ such that $y_j = a_0 + a_1 x_j + a_2 x_j^2 + \cdots + a_m x_j^m$, for $j = 1, \ldots, n$. Here, if $k$ is an integer and $r \in R$, the symbol $kr$ denotes $r + r + \cdots + r$ ($r$ being taken $k$ times) if $k$ is nonnegative and $(-r) + (-r) + \cdots + (-r)$ ($-r$ being taken $-k$ times) if $k$ is negative.

Making use of a result by B. Gordon and T. S. Motzkin [Trans. Am. Math. Soc. 116, 218-226 (1965; Zbl 0141.03002)], the authors derive the following theorem as their main result (Theorem 1): A ring has property IP if and only if it is either a field or the ring with underlying set $\{0, 1\}$, on which addition is modulo 2 and each of its products is 0.

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